11.2

SEQUENCE

$\{(\frac{1}{2})^n\} = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\} \to 0$

CONV $|r|<1$

$\{\frac{1}{n}\} = \{\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \ldots\}$

CONV

$\{2^n\} = \{2, 4, 8, 16, \ldots\}$

DIV $|r|>1$

SERIES

$\sum_{i=1}^{\infty} (\frac{1}{2})^i = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$ GEOMETRIC

CONV $|r|<1, r=\frac{1}{2}$

$\sum_{i=1}^{\infty} \frac{1}{i} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ldots$

DIV

$\sum_{i=1}^{\infty} 2^i = 2+4+8+\ldots$ GEOMETRIC

DIV $|r|>1, r=2$

DOES THERE EXIST A SEQUENCE $\{a_n\}$ SUCH THAT $\{a_n\}$ DIV AND $\sum_{i=1}^{\infty} a_i$ CONV ?

NO $\to$ IF $\{a_n\}$ DIV, THEN $\sum_{i=1}^{\infty} a_i$ DIV

<u>TH'M:</u> IF $\sum_{i=1}^{\infty} a_i$ CONV, THEN $\lim_{n\to\infty} a_n = 0$

<u>PROOF:</u> SUPPOSE $\sum_{i=1}^{\infty} a_i = S \in \mathbb{R}$

IE. $\lim_{n\to\infty} S_n = S$

SO $\lim_{n\to\infty} S_{n+1} = S$

$\{S_1, S_2, S_3, S_4, \ldots\} \longrightarrow S$ OR
$\{S_2, S_3, S_4, \ldots\} \longrightarrow S$
AS $n \to \infty$, $n+1 \longrightarrow \infty$
SO $S_{n+1} \longrightarrow S$

SO $\lim_{n\to\infty} (S_{n+1} - S_n) = \lim_{n\to\infty} S_{n+1} - \lim_{n\to\infty} S_n$ IF BOTH LIMITS EXIST ✓

$= S - S$

$= 0$

BUT $S_{n+1} - S_n$

~~$= \sum_{i=1}^{n+1} a_i - \sum_{i=1}^{n} a_i$~~

$= \sum_{i=1}^{n+1} a_i - \sum_{i=1}^{n} a_i$

$= \left(\cancel{\left(\sum_{i=1}^{n} a_i\right)} + a_{n+1}\right) - \cancel{\sum_{i=1}^{n} a_i}$

$= a_{n+1}$

SO $\lim_{n\to\infty} (S_{n+1} - S_n) = 0$

SO $\lim_{n\to\infty} a_{n+1} = 0$

$\uparrow$

$\{a_2, a_3, a_4, \ldots\} \to 0$

$\{a_1, a_2, a_3, a_4, \ldots\} \to 0$

IE. $\lim_{n\to\infty} a_n = 0$

LOGIC: IF P IS TRUE, THEN Q IS TRUE (CONDITIONAL STATEMENT)

ASSUME TRUE

IF Q IS FALSE, THEN P IS FALSE (CONTRAPOSITIVE)

EQUIVALENT / BOTH TRUE OR BOTH FALSE

(INVERSE) IF P IS FALSE, THEN WE KNOW NOTHING ABOUT Q WITHOUT MORE INFORMATION

(CONVERSE) IF Q IS TRUE, THEN WE KNOW NOTHING ABOUT P WITHOUT MORE INFORMATION

IF $\lim_{n\to\infty} a_n \neq 0$, THEN $\sum_{i=1}^{\infty} a_i$ DIV $\leftarrow$ DIVERGENCE TEST

eg. $\sum_{i=1}^{\infty} \cos^{-1}(e^{-i})$

$$\lim_{n\to\infty} \cos^{-1}(e^{-n}) = \lim_{y\to 0} \cos^{-1} y = \cos^{-1} 0 = \frac{\pi}{2} \neq 0$$

$$\lim_{n\to\infty} e^{-n} = \lim_{n\to\infty} (e^{-1})^n = 0$$

$$r = e^{-1} \approx \frac{1}{3}$$

$$|r| < 1$$

SO $\sum_{i=1}^{\infty} \cos^{-1}(e^{-i})$ DIV (DIVERGENCE TEST)

$\uparrow$

RERUIRED

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eg. $\sum_{i=1}^{\infty} \frac{i^2}{i+1}$

$$\lim_{n\to\infty} \frac{n^2}{n+1} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{\overset{\nearrow \infty}{n}}{\underset{\longrightarrow 1}{1+\frac{1}{n}}} = \infty$$

OR USE L'H ON $\lim_{x\to\infty} \frac{x^2}{x+1}$

SO $\sum_{i=1}^{\infty} \frac{i^2}{i+1}$ DIV (DIV TEST)

BUT WHAT ABOUT

$$\sum_{i=1}^{\infty} \frac{1}{i}$$

$\lim_{n\to\infty} \frac{1}{n} = 0$ SO DOES THAT IMPLY $\sum_{i=1}^{\infty} \frac{1}{i}$ CONV

BY DIV TEST ?

NO, DIV TEST DOESN'T APPLY, DOESN'T SAY ANYTHING SINCE "IF" PORTION OF DIV TEST IS FALSE